Compressible Flow

M. Siavashi

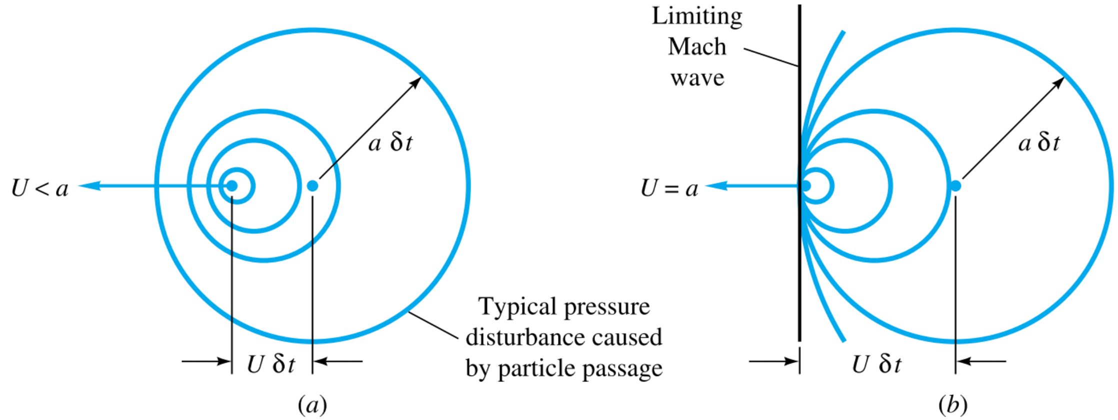
School of Mechanical Engineering

Iran University of Science and Technology

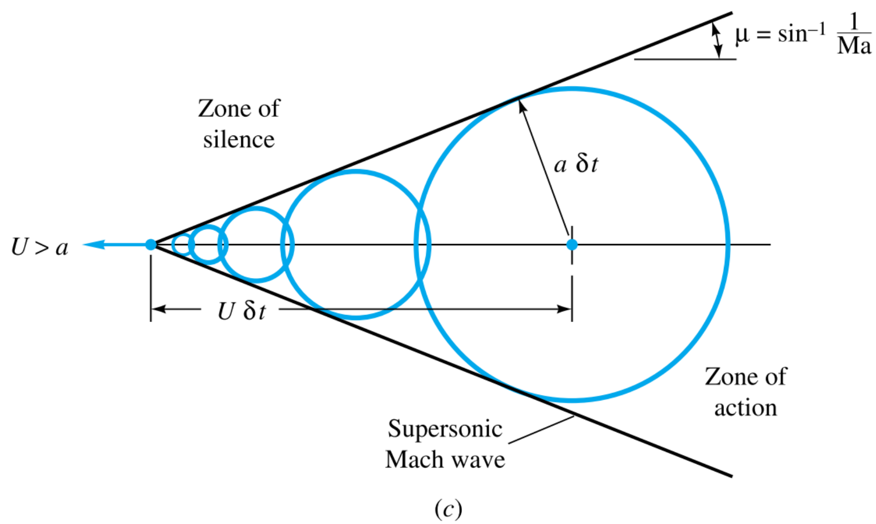
Webpage: webpages.iust.ac.ir/msiavashi
Email: msiavashi@iust.ac.ir
Landline: +98 21 77240391

Fall 2014

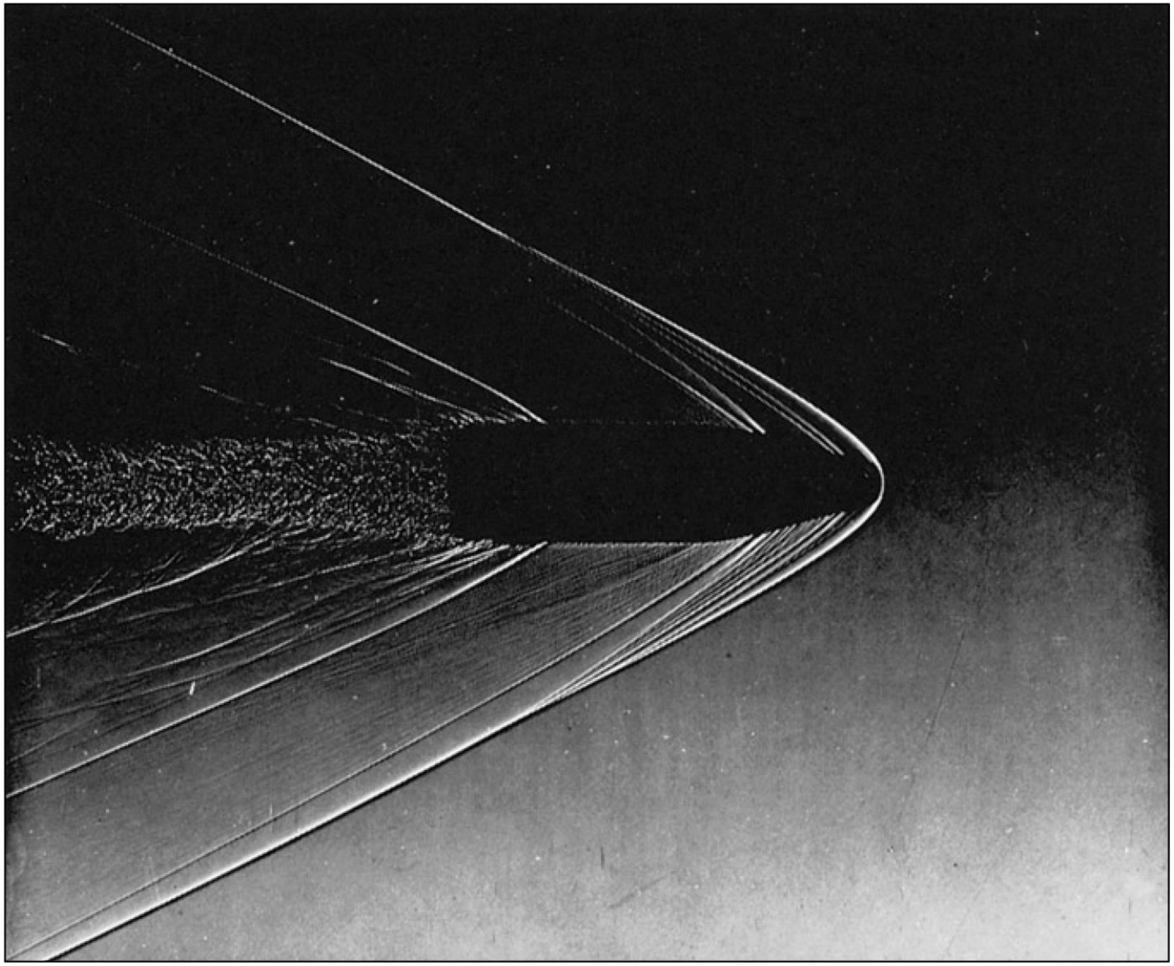
2-D Supersonic Flow



$$\mu = \sin^{-1}\frac{a}{U}\frac{\delta t}{\delta t} = \sin^{-1}\frac{a}{U} = \sin^{-1}\frac{1}{\mathrm{Ma}}$$

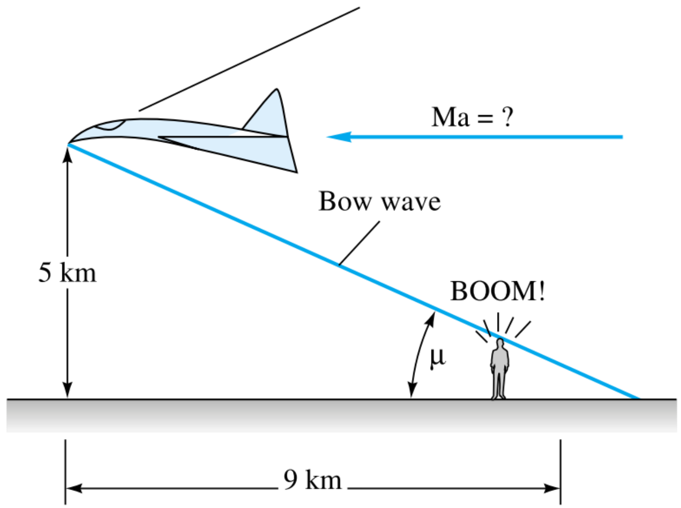


2-D Supersonic Flow



Example

An observer on the ground does not hear the sonic boom caused by an airplane moving at 5-km altitude until it is 9 km past her. What is the approximate Mach number of the plane? Assume a small disturbance and neglect the variation of sound speed with altitude.



$$\tan \mu = \frac{5 \text{ km}}{9 \text{ km}} = 0.5556 \qquad \text{or} \qquad \mu = 29.05°$$

Hence, from Eq. (9.79),

$$\text{Ma} = \csc \mu = 2.06 \qquad \textit{Ans.}$$

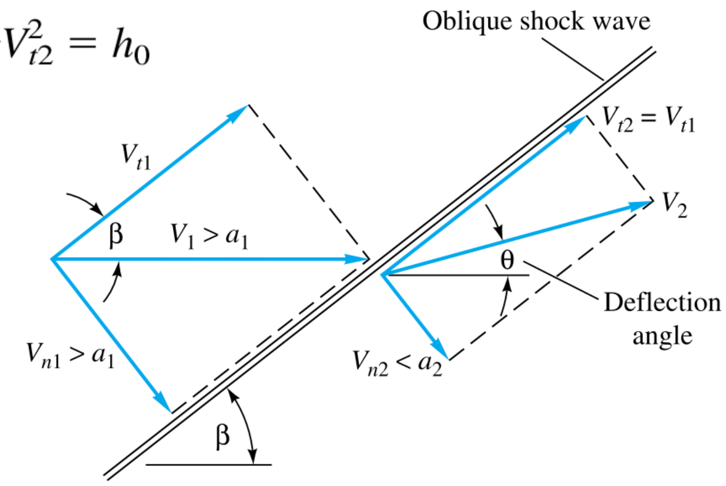
The Oblique-Shock Wave

Continuity: $\rho_1 V_{n1} = \rho_2 V_{n2}$

Normal momentum: $p_1 - p_2 = \rho_2 V_{n2}^2 - \rho_1 V_{n1}^2$

Tangential momentum: $0 = \rho_1 V_{n1}(V_{t2} - V_{t1})$

Energy: $h_1 + \frac{1}{2}V_{n1}^2 + \frac{1}{2}V_{t1}^2 = h_2 + \frac{1}{2}V_{n2}^2 + \frac{1}{2}V_{t2}^2 = h_0$

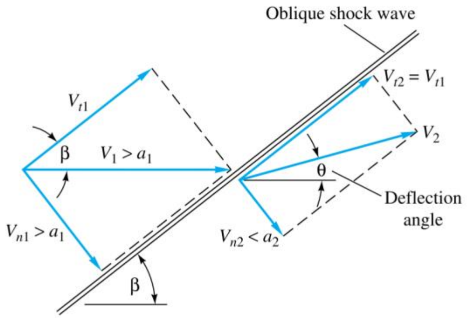


$$V_{t2} = V_{t1} = V_t = \text{const}$$

$$\text{Ma}_{n1} = \frac{V_{n1}}{a_1} = \text{Ma}_1 \sin \beta$$

$$\text{Ma}_{n2} = \frac{V_{n2}}{a_2} = \text{Ma}_2 \sin (\beta - \theta)$$

The Oblique-Shock Wave



$$\frac{p_2}{p_1} = \frac{1}{k+1}\left[2k\,\mathrm{Ma}_1^2 \sin^2\beta - (k-1)\right]$$

$$\frac{\rho_2}{\rho_1} = \frac{\tan\beta}{\tan(\beta-\theta)} = \frac{(k+1)\,\mathrm{Ma}_1^2 \sin^2\beta}{(k-1)\,\mathrm{Ma}_1^2 \sin^2\beta + 2} = \frac{V_{n1}}{V_{n2}}$$

$$\frac{T_2}{T_1} = \left[2 + (k-1)\,\mathrm{Ma}_1^2 \sin^2\beta\right] \frac{2k\,\mathrm{Ma}_1^2 \sin^2\beta - (k-1)}{(k+1)^2\,\mathrm{Ma}_1^2 \sin^2\beta}$$

$$T_{02} = T_{01}$$

$$\frac{p_{02}}{p_{01}} = \left[\frac{(k+1)\,\mathrm{Ma}_1^2 \sin^2\beta}{2 + (k-1)\,\mathrm{Ma}_1^2 \sin^2\beta}\right]^{k/(k-1)} \left[\frac{k+1}{2k\,\mathrm{Ma}_1^2 \sin^2\beta - (k-1)}\right]^{1/(k-1)}$$

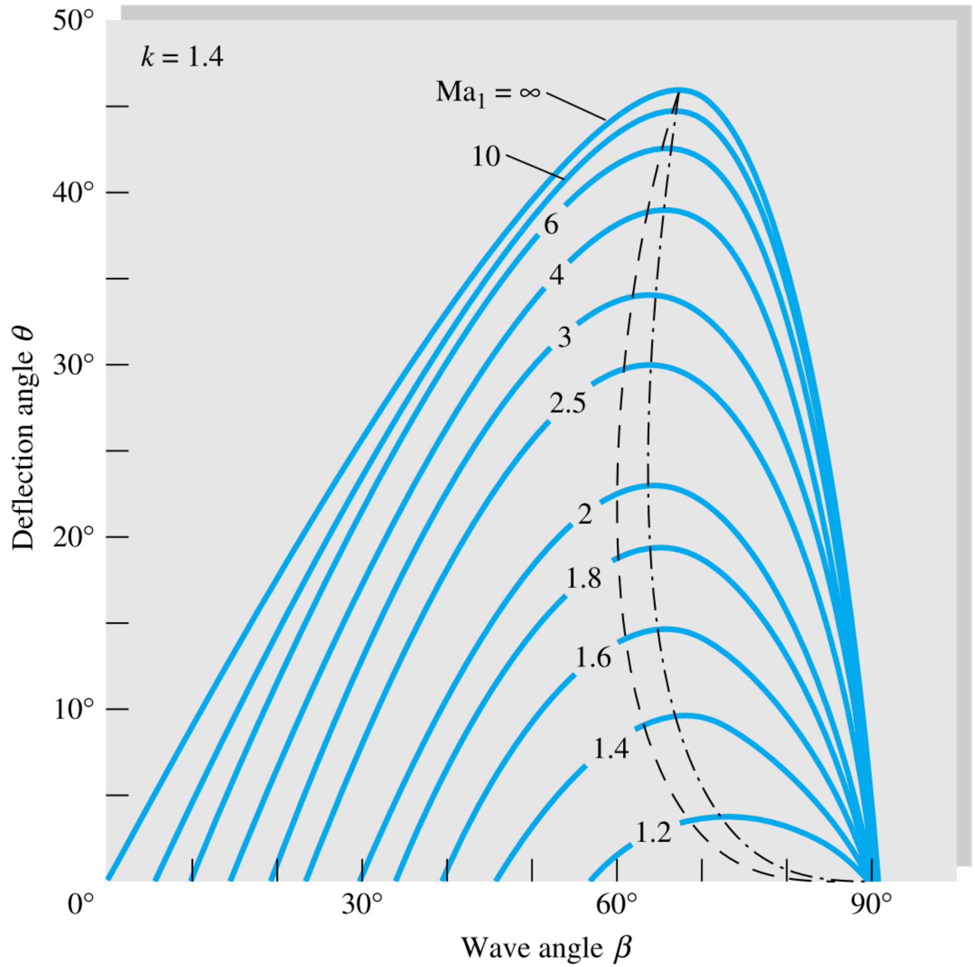
$$\mathrm{Ma}_{n2}^2 = \frac{(k-1)\,\mathrm{Ma}_{n1}^2 + 2}{2k\,\mathrm{Ma}_{n1}^2 - (k-1)}$$

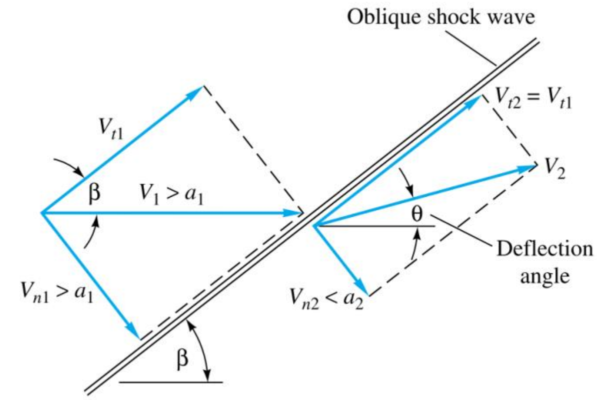
$$\tan\theta = \frac{2\cot\beta\,(\mathrm{Ma}_1^2 \sin^2\beta - 1)}{\mathrm{Ma}_1^2\,(k + \cos 2\beta) + 2}$$

For zero deflections ($\theta = 0$) the weak-shock family satisfies the wave-angle relation

$$\beta = \mu = \sin^{-1}\frac{1}{\mathrm{Ma}_1} \qquad (9.87)$$

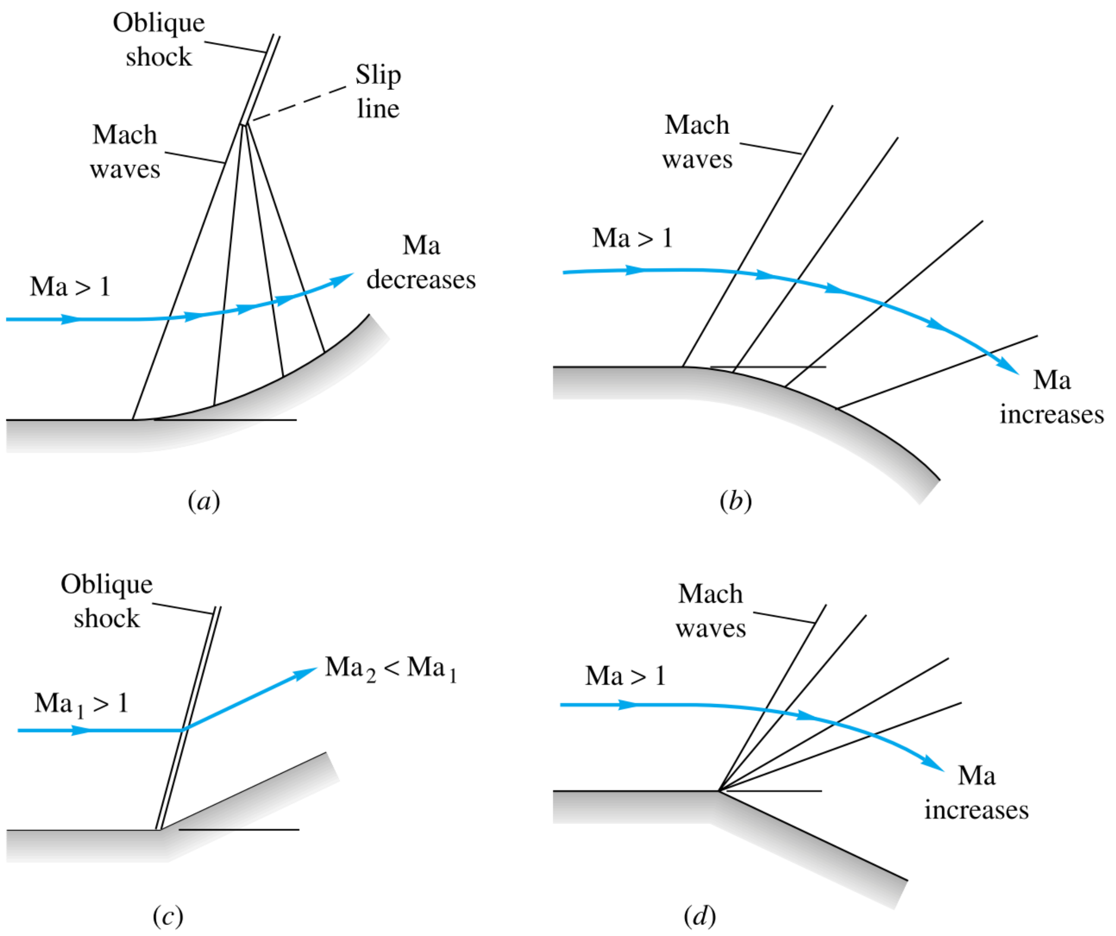
The Oblique-Shock Wave





$$\theta - \beta - M$$

Prandtl-Meyer Expansion Waves



Prandtl-Meyer Expansion Waves

$$\omega(\mathrm{Ma}) = K^{1/2}\tan^{-1}\left(\frac{\mathrm{Ma}^2-1}{K}\right)^{1/2} - \tan^{-1}(\mathrm{Ma}^2-1)^{1/2}$$

$$K = \frac{k+1}{k-1}$$

$$\theta = \omega_2 - \omega_1 = \omega(M_2) - \omega(M_1)$$